ON USE OF AVERAGING IN FxLMS ALGORITHM FOR SINGLE-CHANNEL FEEDFORWARD ANC SYSTEMS

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ABSTRACT

In this paper an active noise control (ANC) algorithm is proposed. This algorithm is based on adaptive filtering with averaging (AFA) and uses a similar structure as that of the FxLMS ANC system. The proposed algorithm, which we call FxAFA algorithm, uses averages of both data and correction terms to find the updated values of the tap weights of the ANC controller. The computer simulations are conducted for single-channel feedforward ANC systems. It is shown that the proposed algorithm gives fast convergence as compared with the FxLMS algorithm, for both broadband and narrowband noise signals. The comparison with the FxRLS algorithm shows that the proposed FxAFA algorithm is a better choice for low computational complexity and stable performance.

1. INTRODUCTION

In contrast to passive noise control (PSN), where absorbing materials are used to "absorb" the unwanted noise signal, the active noise control (ANC) [1] is based on the simple principle of destructive interference of propagating acoustic waves. The concept that acoustic wave interference can be controlled to produce zones of quietness was first proposed by P. Lueg in 1936 for an analogue ANC system. However, success with the early analogue controllers was very limited and in the recent years powerful DSP devices have made possible the development of real time ANC systems with wide range of applications including air conditioning ducts, cars, aircrafts, and so on [2].

The most popular adaptation algorithm used for ANC applications (both broadband & narrowband) is the FxLMS algorithm, which is a modified version of the LMS algorithm [3]. The schematic diagram for a single-channel feedforward ANC system using the FxLMS algorithm is shown in Figure 1. The acoustic path between the reference noise source and the error microphone is called a primary path and is denoted by $P(z)$. The reference noise signal is filtered through the primary path $P(z)$ and appears as a primary noise signal at the error microphone. The objective of the adaptive controller $W(z)$ is to generate an appropriate antinoise signal $y(n)$ propagated by the secondary loudspeaker. This antinoise signal combines with the primary noise signal to create a zone of silence in the vicinity of the error microphone. The error microphone measures the residual noise $e(n)$, which is used by $W(z)$ for its adaptation to minimize the sound pressure at error microphone. Here $\hat{S}(z)$ accounts for the model of the secondary path $S(z)$ between the output $y(n)$ of the controller and that of the error microphone $e(n)$. The secondary path $S(z)$ comprises the digital-to-analog (D/A) converter, reconstruction filter, power amplifier, loudspeaker, acoustic path from loudspeaker to error microphone, error microphone, preamplifier, anti-aliasing filter, and analog-to-digital (A/D) converter. The filtering of the reference signal $x(n)$ through the secondary-path model $\hat{S}(z)$ is demanded by the fact that the output $y(n)$ of the adaptive controller $W(z)$ is filtered through the secondary path $S(z)$ [3].

Although the FxLMS algorithm is computationally simple but its convergence speed is slow and signal dependent. This has motivated researchers to look for fast convergence algorithms. If FIR filter structure is used, the convergence rate can be improved by using variable-step-size LMS, Newton algorithm [3], Kalman algorithm [4], or recursive-least-square (RLS) algorithm. The other approach is to condition the reference signal by employing different filter structures such as lattice filter, subband filter, or orthogonal transform. These different approaches have resulted in a number of ANC algorithms (with improved convergence properties); viz., lattice-ANC system [5], frequency-domain-ANC systems (see [6] and references there in), RLS based algorithms called Filtered-x RLS (FxRLS) [1] and Filtered-x Fast-Transversal-Filter (FxFTF) [7], and IIR-filter-based LMS algorithms called Filtered-u Recursive LMS (FuRLMS) [8], and filtered-v algorithms [9].

The potential instability of IIR filters structures and increased computational demands of other ANC algorithms mentioned above still make FxLMS a good choice for ANC applications. The need for a fast convergence yet a computationally simple algorithm for ANC applications is the main motivation for the investigation conducted in this paper.

Here we explore the realization of an ANC algorithm using adaptive filtering with averaging (AFA). The adaptive filtering with averaging stems from the numerical techniques using

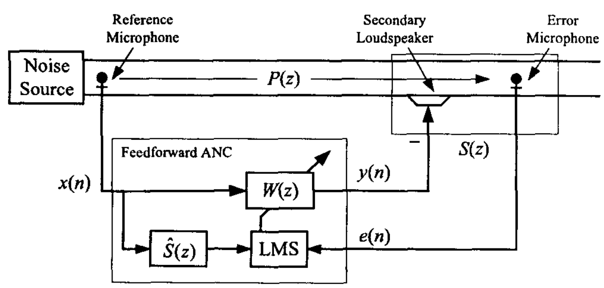

Figure 1. Schematic diagram of a single-channel feedforward ANC system using FxLMS algorithm.

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averaging to accelerate the stochastic approximations (see Ref. [10] and references there in). The simulation results show that this averaging based ANC algorithm, which we call filtered-x AFA (FxAFA) algorithm, provides better results for both broadband and narrowband noise signals.

In Section 2, the proposed FxAFA algorithm is explained in connection with the FxLMS algorithm. Moreover the computational complexity issue is discussed. Sections 3 details the computer experiments performed, and in Section 4 concluding remarks are presented.

2. FxAFA ALGORITHM

In Figure 1 the secondary-path model $\hat{S}(z)$ is obtained offline and kept fixed during the online operation of ANC. The secondary signal $y(n)$ is expressed as

$$y(n) = \boldsymbol{w}^{\mathrm{T}}(n)\,\boldsymbol{x}(n) \tag{1}$$

where $\boldsymbol{w}(n)=[w_0(n)\ w_1(n)\ \cdots\ w_{L-1}(n)]^{\mathrm{T}}$ is the tap weight vector and $\boldsymbol{x}(n)=[x(n)\ x(n-1)\ \cdots\ x(n-L+1)]^{\mathrm{T}}$ is the reference signal picked by the reference microphone. This secondary signal $y(n)$ is filtered through the secondary path $S(z)$ and then combines with the primary noise signal to generate the residual noise $e(n)$. These operations can be expressed as

$$y'(n) = s(n) * y(n) \tag{2}$$

$$d(n) = p(n) * x(n) \tag{3}$$

$$e(n) = d(n) - y'(n) \tag{4}$$

where $y'(n)$ is the secondary signal $y(n)$ filtered through $S(z)$, $d(n)$ is the primary noise signal at the error microphone, $*$ is convolution operation, and $p(n)$ and $s(n)$ are impulse responses of the primary path $P(z)$ and secondary path $S(z)$, respectively. It is important to notice that all these operations, (2)–(4), are carried out internally in the system and we have access only the residual noise signal $e(n)$ being picked up by the error microphone.

It is assumed that there is no acoustic feedback from the secondary loudspeaker to the reference microphone. The FxLMS update equation for the coefficients of $W(z)$ is given as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu\, e(n)\, \boldsymbol{x}'(n) \tag{5}$$

where μ is the step size and $\boldsymbol{x}'(n)$ is the reference signal $\boldsymbol{x}(n)$ filtered through the secondary-path model $\hat{S}(z)$:

$$x'(n) = \hat{s}^{\mathrm{T}}\boldsymbol{x}(n). \tag{6}$$

Here $\hat{s} = [\hat{s}_0\ \hat{s}_1\ \cdots\ \hat{s}_{L-1}]^{\mathrm{T}}$ is impulse response of the secondary-path model $\hat{S}(z)$. In Ref. [10] two averaging based adaptive filtering algorithms are proposed. The first algorithm uses averaging in iterates only and in the second algorithm averaging is incorporated with both iterates and observations. Motivated by the second approach, we incorporate averaging with both the iterates, $\boldsymbol{w}(n)$, and the correction term (the observation vector), $\mu e(n)\,\boldsymbol{x}'(n)$, and propose the following algorithm:

$$\boldsymbol{w}(n+1) = \overline{\boldsymbol{w}(n)} + \overline{\mu\, e(n)\boldsymbol{x}'(n)} \tag{7}$$

where

$$\overline{\boldsymbol{w}(n)} = \frac{1}{n}\sum_{k=1}^{n}\boldsymbol{w}(k) \tag{8}$$

$$\overline{\mu\, e(n)\boldsymbol{x}'(n)} = \frac{1}{n^{\gamma}}\sum_{k=1}^{n}\mu\, e(k)\,\boldsymbol{x}'(k),\quad 1/2 < \gamma < 1. \tag{9}$$

Here computing the running average of the data does not put so much computational burden since averages can be calculated recursively. For example, (8) can be recursively computed as

$$\overline{\boldsymbol{w}(n)} = \frac{1}{n}\Big((n-1)\overline{\boldsymbol{w}(n-1)} + \boldsymbol{w}(n)\Big). \tag{10}$$

Similarly (9) can be computed as

$$\overline{\mu\, e(n)\boldsymbol{x}'(n)} = \frac{1}{n^{\gamma}}\Big((n-1)^{\gamma}\ \overline{\mu\, e(n-1)\boldsymbol{x}'(n-1)} + \mu\, e(n)\,\boldsymbol{x}'(n)\Big). \tag{11}$$

Referring to the feedforward ANC marked in Figure 1, (1), (6), (7), (10), and (11) are combined to give the proposed FxAFA algorithm. We see that the introduction of averaging in the FxLMS update equation results in a multistep algorithm (proposed FxAFA algorithm). Hence an increased computational burden is expected as discussed later in this section. The signal flow diagrams for two algorithms are shown in Figure 2 that depicts that the proposed FxAFA algorithm requires two extra storage bins for previous averaged vectors.

In Section 3 we compare the performance of the proposed FxAFA algorithm with that of FxLMS algorithm and FxRLS algorithm, so for convenience we summarize the FxRLS algorithm here.

$$\boldsymbol{z}(n)= \lambda^{-1}\boldsymbol{\Phi}(n)\ \boldsymbol{x}'(n) \tag{12}$$

$$\boldsymbol{k}(n)= \boldsymbol{z}(n)[1+\boldsymbol{x}'^{\mathrm{T}}(n)\,\boldsymbol{z}(n)]^{-1} \tag{13}$$

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu\, e(n)\,\boldsymbol{k}(n) \tag{14}$$

$$\boldsymbol{\Phi}(n+1)= \lambda^{-1}\boldsymbol{\Phi}(n) - \boldsymbol{k}(n)\,\boldsymbol{z}^{\mathrm{T}}(n) \tag{15}$$

It is important to note that a scalar gain μ is introduced in the update equation of the FxRLS algorithm [7]. The factor λ in (12) and (15) is a "forgetting factor" typically in the range $0.9<\lambda<1$. Here $\boldsymbol{k}(n)$ is an $L\times 1$ gain vector and $\boldsymbol{\Phi}(n)$ is an $L\times L$ inverse correlation matrix. The inverse correlation matrix $\boldsymbol{\Phi}(n)$ is initialized by $\delta^{-1}\mathbf{I}$, where δ is a small positive constant and $\mathbf{I}$ is an $L\times L$ identity matrix.

In Table 1 the computational complexity of three algorithms (FxLMS, FxAFA and FxRLS) is compared on the basis of multiplications required per iteration. In this analysis both ANC controller $W(z)$ and the secondary-path model $\hat{S}(z)$ are assumed to be FIR filters of length L. It is seen that the computational burden of the proposed FxAFA algorithm is between those of the FxLMS algorithm and FxRLS algorithm.

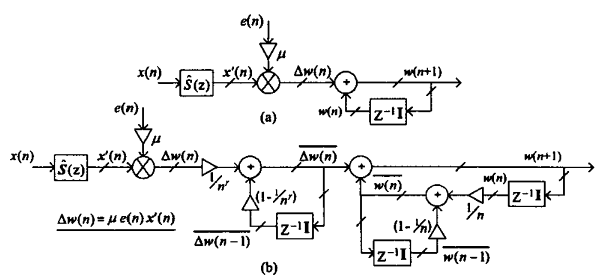

Figure 2. Signal flow diagrams: (a) FxLMS algorithm, (b) FxAFA algorithm.

Table 1. Computational complexity comparison between FxLMS, FxAFA and FxRLS algorithms.

	Number of Multiplications per Iteration		
	FxLMS Algorithm	**FxAFA Algorithm**	**FxRLS Algorithm**
Analytical Expression	$3L+1$	$7L+1$	$3L^2+6L+1$
L=128	385	897	49921
L=256	769	1793	198145

Now we present some comments on the choice of value of γ. For convenience, we rewrite the proposed algorithm in a compact form:

$$w(n+1) = \overline{w(n)} + \alpha(n) \sum_{k=1}^{n} e(k)\, x'(k), \tag{16}$$

where $\alpha(n) = \mu / n^{\gamma}$ is a slowly varying gain parameter. In adaptive algorithms it is desirable to have a large step gain at the startup for fast convergence. As the time increases the gain is desirable to slowly decrease so that the misadjustment is small. The time varying gain $\alpha(n)$ indeed exhibits these properties and $\lim_{n\to\infty} \alpha(n) \to 0$. We observe that $\gamma = 1$ will rapidly decrease the gain but this may reduce update to zero without optimal solution (say w_{opt}) being achieved. Therefore one may wish to choose $\gamma < 1$ [10]. On contrary if $\gamma \to 0$ is selected then $\alpha(n)$ is very slowly decreasing. This is also not desirable for slow convergence. Hence $1/2 < \gamma < 1$ is the recommended range for the values of γ [10].

3. SIMULATIONS

In this section the performance of the proposed FxAFA algorithm is demonstrated using computer simulations. The performance of the proposed algorithm is compared with that of FxLMS algorithm and FxRLS algorithm on the basis of noise reduction R(in dBs) as follows:

$$R(\text{dB}) = -10\log_{10}\left(\frac{\sum e^2(n)}{\sum d^2(n)}\right). \tag{17}$$

The large positive value of R indicates that more noise reduction is achieved at the error microphone. For the primary acoustical path $P(z)$ and the secondary path $S(z)$ the data provided by [1] is used, where both are modeled by IIR filters of order 25. The impulse responses of the primary and secondary paths are shown in Figure 3. The secondary-path model $\hat{S}(z)$ is an FIR filter of order 128, and is identified offline. The adaptive controller $W(z)$ is also an FIR filter of order 128. Since industrial noise often has significant power in the frequency range between 50–250Hz [11], all simulations are carried with the signals having frequency falling in this range. The sampling frequency of 2kHz is used. The parameters for the algorithms are adjusted for fast and stable convergence and are given in Table 2.

3.1 Case 1

Here comparative results of FxLMS, FxAFA and FxRLS are presented for narrowband noise that is a 200Hz sinusoidal signal with additive white Gaussian noise of zero mean and variance

Table 2. Simulation parameters for FxLMS, FxAFA and FxRLS algorithms.

	FxLMS Algorithm	**FxAFA Algorithm**	**FxRLS Algorithm**
	(μ)	(μ, γ)	(μ, λ, δ)
Case 1	1×10^{-4}	1×10^{-2}, 0.6	0.1, 0.999, 0.04
Case 2	1×10^{-5}	1×10^{-3}, 0.6	0.1, 0.999, 5
Case 3	1×10^{-4}	1×10^{-2}, 0.6	0.075, 0.999, 5

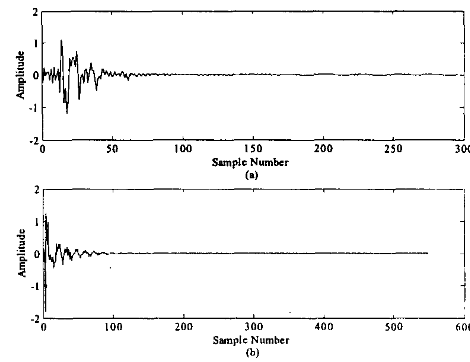

Figure 3. Impulse responses of acoustic paths: (a) Impulse response of primary path $P(z)$, (b) Impulse response of secondary path $S(z)$.

0.05. This signal is used with the feedforward ANC system of Figure 1. Simulation parameters for three algorithms are given in Table 2. The noise reduction curves for performance measure, R, are shown in Figure 4. It is observed that FxRLS algorithm gives fastest convergence speed as compared with those of the FxLMS and FxAFA algorithms, but it diverges towards the higher number of iterations. In Ref. [7] it is suggested that the stability of the FxRLS algorithm can be improved by adding random noise to the input of the FxRLS algorithm. The price paid for the improved stability is the decreased convergence speed [7]. Although the FxLMS algorithm gives stable performance but its convergence speed is slower than that of the proposed FxAFA algorithm.

3.2 Case 2

In this case, the reference noise is a broadband signal and is a sinusoid containing five harmonics (of equal power) with the fundamental frequency of 50Hz. The white Gaussian noise of zero mean and variance 0.05 is added for any measurement noise present. The performance, R, of the proposed FxAFA algorithm is compared with that of FxLMS algorithm and FxRLS algorithm in Figure 5. It is observed that initially the FxRLS algorithm converges very fast but then its convergence speed drops dramatically and finally it diverges again showing the problem of numerical instability. The proposed FxAFA algorithm gives stable and fast convergence as in Case 1.

3.3 Case 3

Here we filter a white Gaussian noise of zero mean and unit variance through a bandpass filter with passband 50–250Hz. The simulation parameters are adjusted for fast and stable performance and are given in Table 2. The performance, R, of FxLMS, FxAFA and FxRLS is presented in Figure 6. It is observed that the proposed FxAFA algorithm gives the best performance of all. The

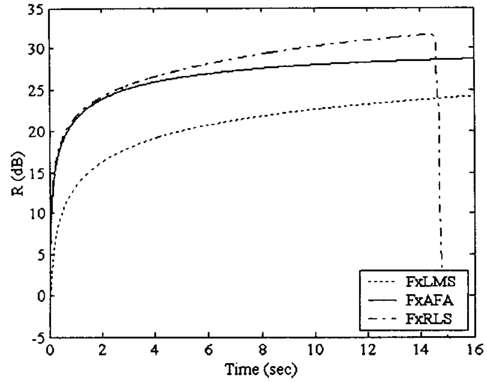

Figure 4. Noise reduction achieved by FxLMS, FxAFA and FxRLS in Case 1.

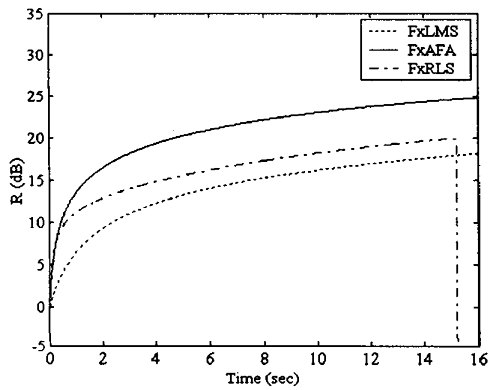

Figure 5. Noise reduction achieved by FxLMS, FxAFA and FxRLS in Case 2.

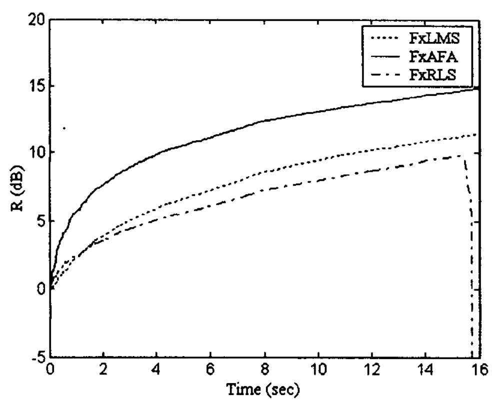

Figure 6. Noise reduction achieved by FxLMS, FxAFA and FxRLS in Case 3.

FxRLS algorithm not only appears as a slowest convergence algorithm but it diverges also towards the higher number of iterations.

4. CONCLUSIONS

Here a new ANC algorithm based on adaptive filtering with averaging is presented. Computer simulations are conducted for single-channel feedforward ANC systems using both broadband and narrowband signals. In comparison to the FxLMS algorithm the proposed algorithm achieves faster convergence, but at the expense of slightly increased computational complexity. It has two adjustable parameters γ and μ, and requires more care in selecting their values. It is seen that by proper choice of γ the larger value for μ can be selected, and thus faster convergence can be achieved.

Experiments conducted with the FxRLS algorithm show that it is not possible to fully stabilize the algorithm and to achieve a fast convergence speed simultaneously. This demonstrates the superiority of the proposed FxAFA algorithm over FxRLS algorithm in both computational complexity and numerical stability.

In this paper offline secondary path modeling is used and it is assumed that the secondary path remains fixed all the time. The development of an ANC algorithm with online secondary path modeling, incorporating adaptive filtering with averaging is a task of future work.

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